

## DESIGN AND IMPLEMENTATION OF DYADIC SHIFT AND GRAPHICAL INVERSE OF STRING SEQUENCES USING REWRITING CYCLIC NORMAL AUTOMATON

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### ABSTRACT

Algorithms that combine numeric and symbolic computing have been of increasing importance and interest over the past decade. The necessity to work with noisy and imprecise data and for speed and accuracy with algebraic and numerical problems encourages the synergy between two types of computing such as numeric and symbolic. In this paper signals are represented as strings or words which can be stored in computer memory, further normal algorithms are written to perform operations like dyadic shift and graphical inverse signals using rewriting cyclic normal automata.

**KEYWORDS:** RCNA, Normal Algorithms, Substitution Formula

### INTRODUCTION

Let  $A$  be an alphabet and  $A^*$  is free monoid. Every algorithm over  $A$  have equivalent normal algorithm. Any algorithm with set of inputs and outputs can be transformed [1] into normal algorithm with same set of input and output pair. This can be realized using normal automata and as well as with family of automata. Every family of automata over a potentially realizable alphabet is normalisable and therefore it is impossible to construct a non normalisable automaton.

Characteristics of family of automata are as follows

- Only basic arithmetic operations are used to accept any input letter.
- Memory used is unbounded which is under the control of automaton [8]
- The set of words which reach any one state of  $F$  from initial state are part of output.
- The input data is stored in internal memory where backtracking is not allowed.
- The acceptance or rejection of any word is depends upon transition rules [9].

### DEFINITION OF IMPORTANT TERMS

#### Rewriting Cyclic Normal Automata

Rewriting cyclic normal automata is triple as  $\langle Q, \text{INI}, \text{FIN} \rangle$  with 4 type of edges where  $Q$  is set of all states, INI is initial state and FIN is final states [12].

#### Normal Algorithm

Normal algorithm consists of set of productions or substitution formulas [15] either simple or terminal type to generate given word in  $A^*$  where  $A$  is an alphabet and  $A^*$  is free monoid.

## Convolution

Convolution is mathematical [14] technique which relates the output  $y[n]$  linear time variant system to input  $x[n]$ . Linear convolution is the process of multiplying any two given non integer sequences of signals.

## Dyadic Shift

In dyadic shift given word has been divided into two parts and shifted such that second part begins the string where first part ends the string.

## Substitution Formula

The formula of the form  $P \rightarrow Q$  used in normal algorithms to derive given string is called as substitution formula. There are two types of substitution formulas [13]

- Simple substitution formula
- Terminal substitution formula

This formula comprises of generic variables, auxiliary variables and string. Terminal formula will be used to end the derivation where simple formula used to replace part of a string by another string while derivation.

## Generic Variables

Variable whose values are taken from the letters [6] of the alphabet are known as generic variable. Generic variables are denoted by the symbols  $\xi$ ,  $\eta$  and  $\mu$ .

## DYADIC SHIFT ON SIGNALS

Let  $w$  be a string from  $A^*$  where  $A$  is an alphabet. The dyadic shift is nothing but [10] dividing the given string into two equal parts and interchanging them in the position. Let  $A = \{w, x, y, z\}$  and  $w = wxyz$  then after dyadic shift  $w = yzxy$ . Let us assume that strings are of even length hence we can divide them into equal parts.

Dyadic shift can be accomplished using left shift  $n/2$  times where  $n$  is length of the given string which is of even number. For example, consider the alphabet  $A = \{w, x, y, z\}$  and the auxiliary alphabet  $E = \{\alpha, \delta, \Gamma\}$ . Let  $\mu$  and  $\xi$  be the generic variables that range over  $A$ , and let  $w = wxyz$  be a word from  $A$ . Normal algorithm for left shift is as follows

**Table 1: Formulas for Dyadic Shift**

Formula	Formula Number	Comments
$\mu\xi\Gamma \rightarrow \xi\Gamma\mu$	0	Move generic variable $\mu$ rightmost of string.
$\alpha\mu \rightarrow \mu\alpha$	1	Move $\alpha$ from left to right of string
$\alpha \rightarrow \Gamma$	2	Replace $\alpha$ by $\Gamma$
$\Gamma \rightarrow \cdot$	3	Move $\Gamma$ right to left using dot
$\rightarrow \alpha$	4	Introduce $\alpha$ at head of given string

The process of finding dyadic shift of  $abcd$  will be completed by applying rule 4, rule 1, rule 2 and rule 3 successively for 2 times because number of characters in a given string is four.

Table 2: Dyadic Shift of Word Wxyz

Step No.	Transformation	Formula Used
1	wxyz	Input string
2	$\alpha wxyz$	4
3	$w\alpha xyz$	1
4	$wx\alpha yz$	1
5	$wxy\alpha z$	1
6	$wxyz\alpha$	1
7	$wxyz\Gamma$	2
8	$wxz\Gamma y$	3
9	$wz\Gamma xy$	3
10	$z\Gamma wxy$	3
11	$zwxy$	3
12	$\alpha zwxy$	4
13	$z\alpha wxy$	1
14	$zw\alpha xy$	1
15	$zwx\alpha y$	1
16	$zwxy\alpha$	1
17	$zwxy\Gamma$	2
18	$zwy\Gamma x$	3
19	$zy\Gamma wx$	3
20	$y\Gamma zwx$	3
21	$yzwx$	Resultant string

The RCNA for the above EPT system is defined by  $C^{DS} = \langle Q, \text{INI}, \text{FIN} \rangle$  where  $Q = \{q_{00}, q_{10}, q_{20}, q_{11}, q_{12}, q_{30}, q_{21}, q_{22}, q_{40}, q_{31}, q_{32}, q_{50}, q_{41}, q_{42}, q_{43}, q_{44}, q_{60}, q_{61}\}$  the set of edges  $T$  where,  $T =$

$(q_{\text{INI}}, \Sigma^*, q_{00})$ ,	$(q_{00}, \Sigma \backslash \xi / \leftarrow, q_{10})$ ,	$(q_{10}, \Sigma \backslash \xi / \leftarrow, q_{20})$ ,	$(q_{10}, / \rightarrow \alpha, q_{11})$ ,
$(q_{11}, \mu / \rightarrow, q_{12})$ ,	$(q_{12}, \Lambda / \vdash, q_{00})$ ,	$(q_{20}, \Sigma \backslash \xi / \leftarrow, q_{30})$ ,	$(q_{20}, \alpha \mu / \mu \alpha, q_{21})$ ,
$(q_{21}, \mu / \rightarrow, q_{22})$ ,	$(q_{22}, \Lambda / \vdash, q_{00})$ ,	$(q_{30}, \Sigma \backslash \xi / \leftarrow, q_{40})$ ,	$(q_{30}, \alpha \Gamma, q_{31})$ ,
$(q_{31}, \mu / \rightarrow, q_{32})$ ,	$(q_{32}, \Lambda / \vdash, q_{00})$ ,	$(q_{40}, \Sigma \backslash \xi / \leftarrow, q_{50})$ ,	$(q_{40}, \mu \beta \Gamma / \beta \Gamma \mu, q_{41})$ ,
$(q_{41}, / \rightarrow \alpha, q_{42})$ ,	$(q_{42}, \alpha \mu / \mu \alpha, q_{43})$ ,	$(q_{43}, \mu / \rightarrow, q_{44})$ ,	$(q_{44}, \Lambda / \vdash, q_{00})$ ,
$(q_{50}, \Sigma \backslash \xi / \leftarrow, q_{60})$ ,	$(q_{60}, \mu / \rightarrow, q_{61})$ ,	$(q_{61}, \Lambda / \vdash, q_{00})$	

The pictorial representation of dyadic shift using RCNA is shown in figure 1

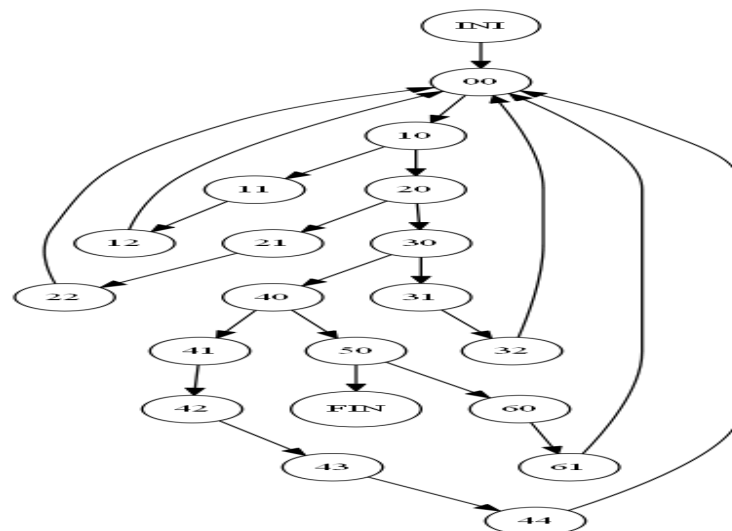


Figure 1: RCNA Diagram for Dyadic Shifting

## GRAPHICAL INVERSE OF GIVEN SEQUENCE

Let  $A$  be an alphabet and  $A^*$  be free monoid which contains set of all strings generated using symbols in  $A$  including null string. Let  $w$  be any string from  $A^*$  to which we have to find graphical inverse. Inverse of a given string can be found by performing left a shift  $n$  time where  $n$  is length of the given string.

For example, consider the alphabet  $A = \{a \ b \ c \ d\}$ , and the auxiliary alphabet  $E = \{\alpha \ \delta \ \Gamma\}$ . Let  $\mu$  and  $\xi$  be the generic variables that range over  $A$ , and let  $w = abcd$  be a word from  $A$ . Normal algorithm for left shift is as follows

**Table 3: Substitution Formulas for Inverse**

Formula	Formula Number	Comments
$\mu\xi\Gamma \rightarrow \xi\Gamma\mu$	0	Move generic variable $\mu$ rightmost of string.
$\alpha\mu \rightarrow \mu\alpha$	1	Move $\alpha$ from left to right of string
$\alpha \rightarrow \Gamma$	2	Replace $\alpha$ by $\Gamma$
$\Gamma \rightarrow \cdot$	3	Move $\Gamma$ right to left using dot
$\rightarrow \alpha$	4	Introduce $\alpha$ at head of given string

The process of finding reverse of  $abcd$  will be completed by applying rule 4, rule 1, rule 2 and rule 3 successively for 4 times because number of characters in a given string is four.

**Table 4: Graphic Inverse of String Abcd**

Step No.	Transformation	Formula Used
1	$abcd$	Input string
2	$\alpha abcd$	4
3	$a\alpha bcd$	1
4	$ab\alpha cd$	1
5	$abc\alpha d$	1
6	$abcd\alpha$	1
7	$abcd\Gamma$	2
8	$abd\Gamma c$	3
9	$ad\Gamma bc$	3
10	$d\Gamma abc$	3
11	$dabc$	3
12	$\alpha dabc$	4
13	$d\alpha abc$	1
14	$da\alpha bc$	1
15	$dab\alpha c$	1
16	$dabc\alpha$	1
17	$dabc\Gamma$	2
18	$dac\Gamma b$	3
19	$dc\Gamma ab$	3
20	$dcab$	3
21	$\alpha dcab$	4
22	$d\alpha cab$	1
23	$dc\alpha ab$	1
24	$dca\alpha b$	1
25	$dcab\alpha$	1
26	$dcab\Gamma$	2
27	$dc\Gamma ba$	3
28	$dcba$	Resultant string

The RCNA for the above EPT system [11] is defined by  $C^{GI} = \langle Q, INI, FIN \rangle$  where  $Q = \{q_{00}, q_{10}, q_{20}, q_{11}, q_{12}, q_{30}, q_{21}, q_{22}, q_{40}, q_{31}, q_{32}, q_{50}, q_{41}, q_{42}, q_{43}, q_{44}, q_{60}, q_{61}, q_{51}, q_{52}, q_{53}, q_{54}, q_{70}, q_{71}\}$  the set of edges  $T$  where,  $T =$

$\{(q_{INI}, \Sigma^*, q_{00}),$	$(q_{00}, \Sigma \backslash \xi / \leftarrow, q_{10}),$	$(q_{10}, \Sigma \backslash \xi / \leftarrow, q_{20}),$	$(q_{10}, / \rightarrow \alpha, q_{11}),$
$(q_{11}, \mu / \rightarrow, q_{12}),$	$(q_{12}, \Lambda / \vdash, q_{00}),$	$(q_{20}, \Sigma \backslash \xi / \leftarrow, q_{30}),$	$(q_{20}, \alpha \mu / \mu \alpha, q_{21}),$
$(q_{21}, \mu / \rightarrow, q_{22}),$	$(q_{22}, \Lambda / \vdash, q_{00}),$	$(q_{30}, \Sigma \backslash \xi / \leftarrow, q_{40}),$	$(q_{30}, \alpha / \Gamma, q_{31}),$
$(q_{31}, \mu / \rightarrow, q_{32}),$	$(q_{32}, \Lambda / \vdash, q_{00}),$	$(q_{40}, \Sigma \backslash \xi / \leftarrow, q_{50}),$	$(q_{40}, \mu \beta \Gamma / \beta \Gamma \mu, q_{41}),$
$(q_{41}, / \rightarrow \alpha, q_{42}),$	$(q_{42}, \alpha \mu / \mu \alpha, q_{43}),$	$(q_{43}, \mu / \rightarrow, q_{44}),$	$(q_{44}, \Lambda / \vdash, q_{00}),$
$(q_{50}, \Sigma \backslash \xi / \leftarrow, q_{60}),$	$(q_{50}, \mu \beta \Gamma / \beta \Gamma \mu, q_{51}),$	$(q_{51}, / \rightarrow \alpha, q_{52}),$	$(q_{52}, \alpha \mu / \mu \alpha, q_{53}),$
$(q_{53}, \mu / \rightarrow, q_{54}),$	$(q_{54}, \Lambda / \vdash, q_{00}),$	$(q_{60}, \Sigma \backslash \xi / \leftarrow, q_{70}),$	$(q_{60}, \mu \beta \Gamma / \beta \Gamma \mu, q_{61}),$
$(q_{61}, \Gamma / \Lambda, q_{FIN}),$	$(q_{70}, \mu / \rightarrow, q_{71}),$	$(q_{71}, \Lambda / \vdash, q_{00}) \}$	

The graphical representation to find inverse is shown figure 2.

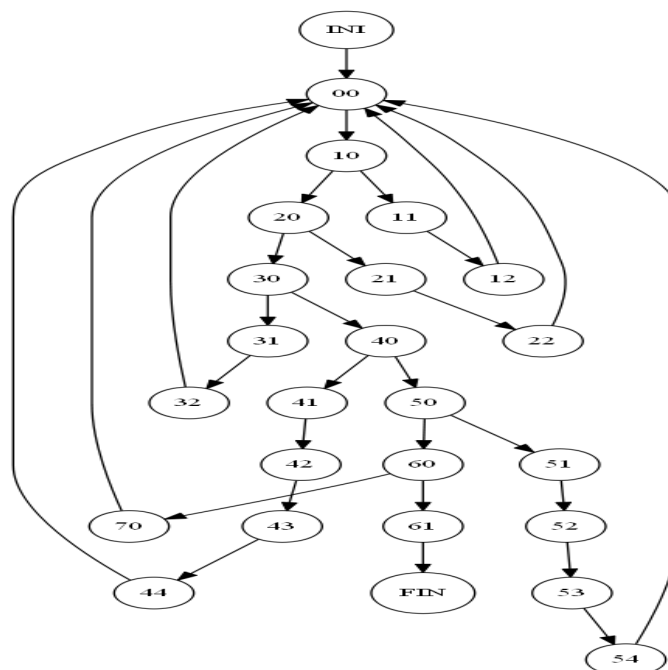


Figure 2: Graphical Inverse Using RCNA

## CONCLUSIONS

In this paper a new technique for performing operations on signals using normal algorithms and RCNA has been represented, which solves the problems like numeric computation of signals, representation of signals in computer process able format. The use of automata in signal processing is still in infancy stage. More efforts are required to explore various methods using automata to implement various signal processing operations. The features of automata will make them as alternative for designing signal operations as conventional methods. In summary, it is considered that RCNA based automata have an excellent potential in signal processing.

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